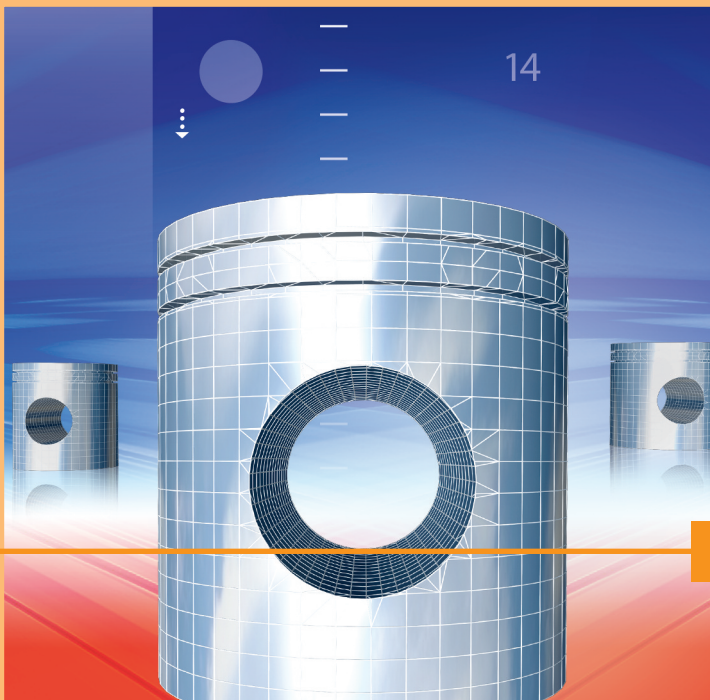


Frank Rieg
Reinhard Hackenschmidt
Bettina Alber-Laukant

Finite Element Analysis for Engineers

Basics and Practical Applications
with Z88Aurora



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The Authors:

Prof. Dr.-Ing. Frank Rieg, Full Professor, Chair for Engineering Design and CAD, University of Bayreuth
Dipl.-Wirtsch.-Ing. Reinhard Hackenschmidt, Senior Academic Councillor, Chair for Engineering Design
and CAD, University of Bayreuth

Dr.-Ing. Bettina Alber-Laukant, Patent Scientist, Academic Councillor, Chair for Engineering Design and CAD,
University of Bayreuth

Translated by the authors with the help of Franziska Auer, Teresa Bertelshofer, Kevin Deese, Christoph Gürtner
and Marlene Süß

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Preface

Following the ongoing strong demand in the last years for an English version of the German standard work “Finite Elemente Analyse für Ingenieure” we decided to satisfy this.

Our aim with this book is:

To provide well-chosen aspects of the finite elements for a student of engineering sciences from the 3rd semester and an engineer already established in the job in such a way that he can apply this knowledge immediately to the solution of practical problems.

Therefore, already in the title of the book we speak of finite element analysis (FEA) and not of finite element method. This gigantic field has left behind the quite dubious air of a method for a long time and today is *the* engineer’s tool to analyse structures. Of course, one can do much more with this process than mechanics: heat flows, electric fields and magnetic fields, actually, differential equations and boundary problems for different fields in general – all of this can be solved with it.

However, everything has begun with the calculation of mechanical structures and, hence, we want to limit ourselves in this book to linear and non-linear statics, stationary heat conduction and natural frequencies. The engineer’s aspect is very substantial to us – it does not appear in the title of this book without any reason: The process was developed fairly “intuitively” in the fifties by airplane engineers for static calculations of airplane structures. It is a process from engineers for engineers!

Hence, we proceed as follows: After a really easy demonstration of the basic procedure, we will discuss the most important points of the elasticity theory, the engineering mechanics and the thermodynamics, as far as the FEA is concerned. With this knowledge we continue with the derivation of the element stiffness matrices. This theoretical knowledge is indispensable for proper and clever working with FEA programs. Then we look at the compilation procedure, at the storage processes and at the solving of the equation systems to calculate the unknowns.

In order to transfer your knowledge into practice, we have put two FE programs on DVD: Z88[®], the open source finite elements program for static calculations, programmed by the lead author of this book, as well as Z88Aurora[®], the very comfortable to use and much more powerful free-ware finite elements program which can also be used for non-linear calculations, stationary heat flows and natural frequencies. Both are full versions with which *arbitrarily big structures* can be computed. The only limits are given by your computer concerning main storage and disc storage and by your powers of imagination. Z88 and Z88Aurora are ready-to-run for Windows,

LINUX, as well as for Mac OS X. For Z88 we directly provide the sources, so that you can study the theoretical aspects in the program code and extend it if necessary. This way, you can also understand the working of memory processes, equations solvers and so forth. Z88 is transparent for the user through input and output via text files. It is a FEA program in the quite classical and original sense. In addition, we think: You only learn the basics with a program like this, as every numerical value can and has to be controlled. As soon as you have understood the basic procedure, you can work with Z88Aurora, which was developed at our *Chair of Engineering Design and CAD* at the University of Bayreuth, Germany, with promotion of the *Oberfrankenstiftung*. Z88Aurora does not take second place in *look and feel* compared to the commercial FEA programs and allows a very professional and contemporary work, directly from CAD data. We do not refer to the known commercial FEA programs here because the versions that are free of charge only offer very limited options concerning the structure sizes with which you could not compute several of the following examples at all. Moreover, we cannot offer source codes for them. In later sections of the book there are many practical examples that we recommend to check. The DVD also contains the input files for all examples. The examples are selected in a way that gradually explains the different aspects of the calculation of structures and mechanical structures.

Furthermore, we have developed an app for Android devices called Z88Tina (www.z88tina.de) which is a very, very small cousin of our full-featured freeware FEA program Z88Aurora (www.z88.de) and is derived from the open source FEA program Z88V14OS. Z88Tina can be downloaded from Google Play Store: <https://play.google.com/store/apps/details?id=z88tina.fr>

For this fourth German edition (and first English edition) we have completely revised our book on finite element analysis: The theoretical section has been extended concerning shell elements (by Prof. F. Rieg, PhD), non-linear calculations (by C. Wehmann, PhD), stationary heat conduction (by M. Frisch, M.Sc.) and natural frequencies (by M. Neidnicht, PhD). The examples have been strongly extended and updated. Our employees M. Frisch, M.Sc., M. Neidnicht, PhD, F. Nützel, M.Sc., C. Wehmann, PhD, J. Zapf, PhD, and M. Zimmermann, M.Sc., did the programming and testing of Z88Aurora version 2 and gave valuable recommendations for the text of this book. We wish to thank them all a lot. Our very special thanks is directed towards Kevin Deese and Christoph Wehmann for their systematic translation error search. It was a hell of a work. We also thank our publishing house Carl Hanser Verlag for the exemplary realization of this book.

The work on this book was again a pleasure to us and we hope you will enjoy this book.

Frank Rieg, Reinhard Hackenschmidt and Bettina Alber-Laukant
Bayreuth, Germany, June 2014

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The DVD that comes with the book *Finite Element Analysis for Engineers* contains the program versions Z88 V14 OS and Z88Aurora V2 including all data necessary to use the examples of both versions. The content of the DVD is organized as follows:

<i>/z88_examples_z88aurora/:</i>	Examples for Z88Aurora V2
<i>/z88_examples_z88v14os/:</i>	Examples for Z88 V14 OS
<i>/z88aurora/:</i>	Installer and documentation Z88Aurora V2
<i>/z88v14os/:</i>	Unzipped directories Z88 V14 OS

Installation of Z88 V14 OS

Z88 V14 OS is available as a ready-to-run version as well as a version for self-compiling in the directory */z88v14os/* for the following operating systems:

- 32 BIT Windows
- 64 BIT Windows
- 32 BIT LINUX
- 64 BIT LINUX
- 64 BIT Mac OS X

In the file *z88mane.pdf* in the directory */z88v14os/docu/* you find the detailed documentation for installation and compiling.

Installation of Z88Aurora V2

Z88Aurora V2 is available in the directory */z88aurora/* as installer for

- 32 BIT Windows and
 - 64 BIT Windows
- and as TAR.GZ for
- 64 BIT LINUX Suse 12.1 and 12.2
 - 64 BIT LINUX Ubuntu 11.04, 12.04 and 14.04
 - 64 BIT Mac OS X ex 10.6 (Please note that when using UNIX und Mac the access rights have to be adapted.)

In the directory */z88aurora/installer/* you find the detailed installation manual for the corresponding operation system.

Please note, that when using Mac OS X the GTK+-package *gtk+4z88.dmg* (which you find in the directory */z88aurora/installer/macosx*) has to be installed at first.

In the directory */z88aurora/docu/* you find the theory manual and the user guide.

Software Updates

The DVD's software status is June 10th, 2014.

On www.z88.de you can find the user forum as well as updates and error corrections.

